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THEORY OF THE ACTION OF PUMPS.

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To be read Friday, May 25th, 1894.

This paper is, in the main, a compilation from scattered memoirs (mostly German) on the subjects in question, and, save for incidental novelties of treatment, contains nothing original.

It is offered to this Society first, because the secretary informed the author last week he would be pleased to have a paper for this evening; and secondly, because Past President Mr. Kennedy thought it would be of value to have Bach's celebrated experiments recorded in the Society's proceedings. They are accordingly embodied in Sect. III. It is proposed to give an account of Bach's and Riedler's later experiments on pumps themselves to the Society in a future paper.

The paper is divided into three parts: The first part is devoted to the consideration of the forces acting and resistances experienced during the suction stroke of a single acting pump without and with a vacuum-vessel; the second part deals with the nature and action of valves; and the third gives the results of Bach's experiments as above mentioned.

Parts IV, V, and VI, to be offered at a future time, will contain the investigation of the delivery stroke, the experiments of Riedler and Bach on pumps leading to the determinations of the limiting speed of pumps, and some suggestions as to the direction for future experiments on the subject.

The symbols used in this paper are as follows:

G = Weight of one cub. ft. of fluid to be pumped.

g = acceleration due to gravity.

A = area of pump bucket or plunger.

a = area of suction pipe.

v = speed of plunger at any instant.

v = acceleration of same at same instant.

u = speed of water in suction main at same instant.

u = acceleration of water in suction main at same instant.

l = length of suction main.

l_1 = length of suction main between pump chamber and vacuum-vessel.

l_2 = length of suction main from vacuum vessel to suction well.

h_s = vertical distance from water level in well to bottom of stroke.

x = distance of plunger from bottom of stroke at instant considered.

S = centre of stroke.

B = pressure in pounds per sq. in. corresponding to barometric pressure.

F_1 = average force required to act on the plunger in order to overcome weight.

F_2 = average force required to act on the plunger in order to overcome inertia.

F_3 = average force required to act on the plunger in order to overcome friction.

The work done by the atmosphere while forcing the fluid up the lift-main during the suction stroke consists of three parts: that required to raise the water merely against gravity; that required to overcome the force of inertia; and that required to be expended on prejudicial hydraulic resistances.

Part I.—The Suction Stroke.

SECTION 1.—WORK AGAINST GRAVITY.

The work expended per stroke in merely raising the water may be estimated as follows:

The force acting on the suction pipe is $G\,a\,(h_s + x)$, and acts through a distance $\frac{A}{a}dx$ when the plunger rises the amount dx. The work done during the upward stroke is therefore

$$F_1\,S = \int_0^S G\,A\,(h_s + x)\,dx = G\,A\,S\left(h_s + \frac{S}{2}\right) \ldots\ldots (1)$$

SECTION 2.—WORK AGAINST INERTIA FORCES.

If the water in the suction pipe is at rest, and then receives a velocity u, an amount of work equivalent to $\frac{Mu^2}{2g}$ foot lbs., the kinetic energy of the mass of water M, must be expended. If it have initially a velocity u_1 and this is to be increased to u_2, the amount of work to be expended must be the equivalent of the difference of the kinetic energies before and after alteration of speed of the water; or in symbols

$$\frac{M\,(u_2^2 - u_1^2)}{2g}.$$

Should u_1 be greater than u_2, work need not be expended upon, but will be done by the fluid; and if $u_2 = u_1$, no work need be done.

In applying these principles, there are three different masses of water, the effects of whose inertia are to be considered.

(a) A weight of water $G\,A\,dx$ which enters the suction pipe from the well while the plunger describes the space dx, and receives the velocity u which obtains in that pipe at the instant considered. A work in foot lbs. equivalent to $\frac{G\,A\,u^2\,dx}{2g}$ is necessary to effect this; and can be performed by a force $\frac{G\,A\,u^2}{2g}$ moving through a distance dx.

(b) The weight of water $G\,a\,l$ which is contained in the suction pipe of a single acting pump without a vacuum vessel which is at rest at the beginning of the stroke.

If there be a vacuum vessel, the water contained in that part of the suction pipe between this and the well is continuously in motion, while that part which lies between pump chamber and vacuum vessel comes to rest during the return stroke. If the piston acceleration be $\dot{i}$ and that of the fluid in the suction pipe $\dot{u}$, then the work $\frac{G\,a\,l\,\dot{u}\,dx}{g}$ must be expended while the plunger describes the distance dx. This is performed by a force $\frac{G\,a\,l\,\dot{u}}{g}$

(c) The weight of water $G\,A\,(x + e\,S)$ contained at the moment in the pump chamber must also be accelerated, and an amount:— $\frac{G\,A\,(x + e\,S)\,\dot{i}\,dx}{g}$ foot lbs. of work must be expended while the plunger travels the distance dx.

The force $\frac{G\,A\,(x + e\,S)\,\dot{i}}{g}$ acting through dx will accomplish this.

At the instant considered, then, there is required an amount of work to overcome the inertia of the three masses of fluid, as follows:

$$\left[\frac{G\,A\,u^2}{2g} + \frac{G\,a\,l\,\dot{u}}{g} + \frac{G\,A\,(x + c\,S)\,\dot{v}}{g}\right]dx$$

or since $A\,\dot{v} = a\,\dot{u}$.

$$\frac{G\,A}{g}\left[\frac{u^2}{2} + (l + x + c\,S)\,\dot{v}\right]dx.$$

The work done during the whole stroke is therefore

$$F_2\,S = \frac{G\,A}{g}\int_0^s\left[\frac{u^2}{2} + (l + x + c\,S)\,\dot{v}\right]dx \;\ldots\ldots\ldots(2)$$

an integral which can be easily found for certain kinds of motion of the plunger, as we shall see.

The force which must at any instant be exerted on the fluid during the suction stroke is obviously

$$\frac{G\,A}{g}\left[\frac{u^2}{2} + (l + x + c\,S)\,\dot{v}\right]$$

where u, x, and $\dot{v}$ are variables.

SECTION 3.—WORK AGAINST HYDRAULIC RESISTANCE.

The prejudicial hydraulic resistances acting during the suction stroke occur: at the entrance to the suction pipe; at the foot valve, if any; by pipe friction and variation of section or direction in the suction pipe and the pump chamber; and at the suction valve.

If h_r be the total head lost in resistances, then a force $G\,a\,h_r$ must be exerted on account of these resistances; and since, while the plunger travels a distance dx, this force will have to move through $\frac{A}{a}\,dx$, the work then done will be

$$G\,a\,h_r\,\frac{A}{a}\,dx = G\,A\,h_r\,dx$$

h_r may also be put equal to $\zeta\frac{u^2}{2g}$; where ζ is an experimental constant, and is made up of the following five component parts:

- ζ_1 for entrance to suction pipe.
- ζ_2 for flow through foot valve.
- ζ_3 for flow through suction pipe.
- ζ_4 for flow through suction valve
- ζ_5 for motion in pump chamber.

The work required to overcome these resistances may thus be written:

$$G\,A\,(\zeta_1 + \zeta_2 + \zeta_3 + \zeta_4 + \zeta_5)\frac{u^2}{2g}\,dx \qquad \text{for the travel } dx,$$

and for the whole stroke:

$$F_3\,S = G\,A\left(\Sigma(\zeta)\right)\int_0^s\frac{u^2}{2g}\,dx \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

SECTION 4.

The pressure of the external atmosphere must be capable of overcoming all these opposing forces at every moment; or at all events the work done during the stroke by the atmosphere must be at least equal to the sum of all the opposing quantities of work. The distance moved through by this atmospheric force during the suction stroke S is $S\frac{A}{a}$; so that if B be the pressure in pounds per square inch corresponding to the height of the barometer, we must have

$$G\,B\,a\,S\,\frac{A}{a} \geq (F_1 + F_2 + F_3)\,S$$

$$\text{or } G\,B\,A\,S \geq G\,A\,(h_s + S)\,S$$

$$+ \frac{G\,A}{g}\int_0^s\left[\frac{u^2}{2} + (l + x + c\,S)\,\dot{v}\right]dx$$

$$+ G\,A\left(\zeta_1 + \zeta_2 + \zeta_3 + \zeta_4 + \zeta_5\right)\int_0^s\frac{u^2}{2g}\,dx \;\ldots(4)$$

The force GAB must at every moment be equal to or greater than

$$GA\,(h_s + x) + \frac{GA}{g}\left[\frac{u^2}{2} + (l+x+cS)\,\dot{v}\right] + \frac{GA}{2g}\left[\Sigma\,(\zeta)\right]u^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Obviously $Av = au$; and $A\dot{v} = a\dot{u}$ if the law of continuity holds.

The condition expressed in (5) is best investigated graphically; and for this purpose we may write it thus

$$B - \left[(h_s + x) + \left(\frac{u^2}{2g} + \frac{l + x + cS}{g}\dot{v}\right) + \Sigma\,(\zeta)\,\frac{u^2}{2g}\right] \geqq 0 \qquad \ldots\ldots\ldots(5a)$$

or

$$B - \left[(h_s + x) + \frac{l + x + cS}{g}\dot{v} + \Big(1 + \Sigma(\zeta)\Big)\frac{u^2}{2g}\right] \geqq 0 \qquad \ldots\ldots\ldots(5a)$$

In Fig. 2 let AB represent the stroke of the plunger and let A be the lowest position. We may now shew graphically the values of the terms in expression ($5a$) by setting up from AB their numerical amounts expressed in feet of water; positive quantities upwards, negative downwards.

The constant atmospheric pressure is first represented by the straight line drawn so that $AJ = BK = B$. The lift $(h_s + x)$ will be represented by the slightly inclined line, when $AG = h_s$, and $BH = (h_s + S)$. The term involving $\dot{v}$ the acceleration of the plunger will be shown by such a curve as CDE, while that involving u^2 and composed of resistances at entrance and during transit through the suction pipe by the graph AFB.

For any given piston position, as Q: if MR be taken equal to the sum of QS, QP, and QO: then MQ represents the excess of the force due to atmospheric pressure over the resisting forces of gravity, inertia, and hydraulic resistance. If LN be the locus of all such points as M, it represents the resultant curve of pressure on the under side of the plunger. So long as all the points in LN lie above AB, ($5a$) is satisfied. If the curve intersects AB as in Fig. 3, the plunger leaves the water behind as at T: and is struck a blow by it when it catches up again after U.

In Fig. 4, the acceleration force is so great that the water is unable, even from the beginning of the stroke, to follow up the plunger. The smaller the lift the less is the fear of the suction column breaking. The greater a, the area of the suction pipe, the smaller the importance attaching to the curves CDE and AFB; for the quantities they represent diminish as the square of the increase of a. Hence the speed of the pump may be increased by increasing the diameter of the suction pipe. The greater l is, the length of the lift main, the smaller must be the speed of the plunger. With high piston speed and a not very excessive pressure in the force main, the pressure in the pump chamber may, towards the end of the stroke, become so great as to open the discharge valve and force water to flow through, before the plunger gets to the top of the stroke. This is represented in Fig. 5.

In drawing these curves for a crank driven pump, it is usually sufficiently accurate to neglect the obliquity of the connecting rod, when CE becomes a straight line, and AFB a curve of sines, as shown in Fig. 6.

From the above considerations and diagrams, it is seen that a considerable portion of the available atmospheric head is required at the beginning of the suction stroke for the purpose of merely accelerating the mass of water in the lift main. This in fact determines the maximum plunger speed with given values of the pump dimensions; and for long lift mains the allowable speed must be very small.

To overcome this difficulty, a vacuum vessel is inserted in the main in order to insure an inflow from the well as nearly steady as possible. The mass of water to be accelerated at the commencement of the suction

stroke is then reduced to that between this vessel and the plunger; so that instead of l in the above equations we must insert l (the distance between vacuum vessel and plunger), which can usually be made very small.

If the vacuum vessel be sufficiently large, the pressure in it will alter but little during the double stroke, so that the motion of the entering fluid between it and the well becomes very approximately uniform.

The work expended on the inertia of the water entering the suction pipe is again given out at the vacuum vessel; and then is effective in accelerating the fluid between it and the pump chamber, when it comes to rest at the end of the down stroke.

The work necessary to overcome the inertia of the water entering the suction pipe with uniform velocity u'' is obviously $\frac{GAS\,(u'')^2}{2g}$

If l_2 be the length of lift main from vacuum chamber to well, the work spent against hydraulic resistance will be $GAS\,\zeta''\frac{(u'')^2}{2g}$; where $\zeta'' = \zeta_1 + \zeta_2 + \zeta''_{2,1} + \zeta''_3$ refers to the length l_2

In the length l_1 between vacuum vessel and pump chamber, an amount of work

$$GAS\,\zeta \int_o \frac{(u')^2}{2g}\,dx$$

must be spent; since here u' is variable.

Also here $\zeta' = \zeta'_1 + \zeta'_1 + \zeta_4 + \zeta_5$ where ζ'_1 is the coefficient of resistance at the entrance to the pipe leading from vacuum to pump chamber; ζ'_3 for the friction in the latter corresponding to the length l_1.

When a vacuum vessel is fitted, we have only to change equ. (5 *a*) into the expression

$$B - \left[(h_s + x) + \frac{l_1 + x + c\,S}{g}\,\dot{v} + (1 + \zeta'')\frac{(u'')^2}{2g} + \zeta'\frac{(u')^2}{2g} \right] \geqq o \qquad (5\,b)$$

Part II.—Valves.

In this paper we concern ourselves only with automatic valves: those, *i.e.*, which open and close under the influence of fluid pressure.

During the lifting of the valve and after completion of the stroke, water flows through the opening so made, by reason of the difference of pressures above and below the valve, and keeps it open. If now the velocity of the water diminishes to nothing, the valve should gradually close, and should touch its seat exactly at the moment when the speed of the water becomes nothing; otherwise a return flow of water will take place through the unclosed valve for an instant. This is to be avoided not only because of the diminution of delivery thereby occasioned, but also on account of its effect in destroying smoothness of working.

If it be the suction-valve which closes too late, the plunger will have described a short distance x of its return stroke before the valve touches its seat. When this does happen, the discharge valve has to be struck open and the mass of water on the other side of it accelerated; and since the acceleration is a maximum with crank driven pumps at the end of the stroke, the effect of this description of even a small value of x will produce a blow in the pump chamber, which will be the more severe the later the suction valve closes and the greater the mass of water to be accelerated.

A similar, if not usually as great, effect is produced when the delivery valve closes after the plunger has begun to perform its return stroke.

In order to obtain a timeous closing of the valves other forces must be introduced besides those of fluid pressure; and three kinds of valves may be distinguished:

(*a*) *Gravity loaded valves*, when gravity alone acts;

(*b*) *Spring loaded valves*, when the elasticity of the valve itself or of another body is employed; and

(c) ***Gravity and spring loaded valves***, when both kinds of force are essential.

A valve spring loaded only properly speaking only exists when the specific gravity of the valve is unity.

Gravity can often be made to effect correctly timed closure; but when heavy valves are in question with large inertia effects recourse must be had to springs.

SECTION 1.—GRAVITY VALVES.

At the moment when the valve begins to close, its weight in water, W, must be a little greater than P, the difference of the forces exerted on its sides by the fluid streaming past it, which we shall call the *valve head.*

P is a function of (see Fig. 7) a_1, a_2 (the upper and lower areas); p_1, p_2; the lift h, the velocity u, with which the water is flowing through d, at the beginning of closure; of the size and shape of the valve box, and of construction and finish of the valve.

The pressure in the fluid between valve and seat is not known at present; but assuming it to be but little different from p_2, and that the valve box is large enough to have no influence on P, we may write with sufficient accuracy for the valve head

$$P = (p_1 - p_2)\, a_1 + k \frac{u_1^2}{2g} Ga_1 \ldots\ldots (11)$$

This may be shown as follows:

The force tending to keep up the valve is the difference of the lower and upper pressures acting on the area of underside of valve, together with the force required to change the momentum of the water flowing through.

In Fig. 8, if u_1 and v represent the initial and final velocities of the water passing the valve, the change of velocity is represented by u.

If a mass of water M impinge against the valve per second, the force required to give this a velocity u in a second is $F = \frac{Mu}{g}$ lbs. The vertical component of this force is

$$F_v = \frac{Mu}{g} \cdot \frac{u_1}{u} = \frac{Mu}{g} = \frac{Mu}{g}$$

In other words, the force acting on this account on the valve is that required to destroy the vertical speed of the water passing. Now,

$$M = Ga_1 u_1.$$

Hence $F_v = n \frac{Ga_1 u_1^2}{g}$ where n is a co-efficient allowing for friction, etc. Putting $K = 2n$ we may write:—

$$F_v = K \frac{u_1^2}{2g} Ga_1$$

which justifies the form of (11), K being a co-efficient to be determined by experiment, and depending for its value upon the final direction of the stream, the breadth of the valve seat, and nature of the valve.

The greater the weight of the valve the more readily it will close; on the other hand, the less it will lift at maximum flow, and the greater resistance will it offer to the passage of the water. When the distance from valve to well is great, prompt closure must on this account be somewhat sacrificed in order to diminish the prejudicial hydraulic resistance. A return flow is then inevitable through the valve; and what must be carefully attended to is the securing of as direct and energetic an action on the upper side of the valve due to this return flow as possible.

Such an arrangement as shown in Fig. 9, where the water is discharged through the side channel A, and where the return flow from A rather tends to lift or jam the valve than to close it, is by all means to be avoided.

The kinetic energy per pound of the water passing between valve

and seat is $\frac{v^2}{2g}$ if v is its velocity there. Now this kinetic energy consists of $k\frac{u_1^2}{2g}$ that part of $\frac{u_1^2}{2g}$ which it still retains, and $\frac{p_1 - p_2}{G}$ the head equivalent to the difference of pressures below and above.

$$\text{hence } v = C_v \sqrt{k\,u_1^2 + 2g\frac{p_1 - p_2}{G}}$$

where C_v is the coefficient of velocity.

Assuming the co-efficient of contraction in the area a_1 under the valve to be unity, we shall therefore have

$$\left[a_1 u_1 = c\,qh\sqrt{\left[k u_1^2 + 2g\frac{p_1 - p_2}{G}\right]}\right] \ldots\ldots$$

Where c is the co-efficient of discharge and $q = \pi d_1$

Hence

$$p_1 - p_2 = \left[\left(\frac{a_1 u_1}{c\,q\,h}\right)\right]^2\frac{G}{2g} - \frac{k\,u_1^2}{2g}G$$

$$= G\frac{u_1^2}{2g}\left[\left(\frac{a_1}{c\,q\,h}\right)^2 - k\right.$$

Substituting in (11) the valve head is

$$P = Ga_1\frac{u_1^2}{2g}\left[\left(\frac{a_1}{c\,q\,h}\right)^2 + K - k\right]$$

Or putting f for $K - k$, the weight of the valve in the fluid,

$$W \underset{>}{=} P = Ga_1\frac{u_1^2}{2g}\left[\left(\frac{a_1}{c\,q\,h}\right)^2 + f\right] \ldots\ldots\ldots(12)$$

SECTION 2.—SPRING AS WELL AS GRAVITY VALVES.

1. *Opening.*—At the instant of opening the forces acting on the valve are $a_1 p_1 + a\,p$ upwards (where $a = \frac{\pi}{4}\left(d_2^2 - d_1^2\right)$ and p is the mean pressure in the space between valve and seat); and $a_2 p_2 + W + F$ downward (where F is the force due to the spring; and W the valve's weight in water).

The equation of motion of the valve is therefore

$$a_1\,(p_1 - p_2) - a\,(p_2 - p) - W - F = \frac{W + G\,V}{g}\,a = \frac{W}{g}\frac{\gamma}{\gamma - 1}\,a$$

Where a is the accelerations and V the volume of the valves and γ is the specific gravity.

Hence if $\frac{W}{g}\frac{\gamma}{\gamma-1}$ be denoted by M; the following is an expression for the acceleration :

$$a = \frac{a_1\,(p_1 - p_2)}{M} - \left[\frac{a}{M}(p_2 - p) + \frac{\gamma - 1}{\gamma}g + \frac{F}{M}\right] \ldots\ldots(13)$$

From which it is seen that the acceleration increases with difference of pressures $(p_1 - p_2)$; and diminishes as the mass of the valve, the spring loading, and the area of valve seat increase. The last only so long as $p_2 - p$ is positive, which is probably the case with high speed pumps as a vacuum must obtain for an instant when the valve lifts quickly at first.

Assuming as small a value as possible for $p_1 - p_2$ (since the smaller this is, the greater may the suction head be), and constant, the time required to lift the valve a given amount will be shorter the smaller its mass, the less the initial spring-load, and the smaller the valve seat area.

The conditions for equilibrium of the valve are obtained by putting $a = 0$ in (13); then

$$a_1\,(p_1 - p_2) - a\,(p_2 - p) - W - F = 0$$

$$\text{or } p_1 - p_2 = \frac{a}{a_1}(p_2 - p) + \frac{W + F}{a} \ldots\ldots\ldots(14)$$

$p_1 - p_2$ may be called the *valve load.*

Section II.—Hydraulic Resistances when open.

The hydraulic resistances opposed by the valve to the passage of the fluid, the sum of which we call the *valve resistance*, may be enumerated as follows :

That caused (*a*) by the change of direction of the stream.

(*b*) by the change of velocity occasioned by passing from area a_1 to area hq.

(*c*) by friction against the underside of the valve and valve seat.

(*d*) by change of direction and sectional area in the valve box.,

and (*g*) in the case of a valve guided from below, by the changes of direction, and sectional area of stream as well as the friction against the surfaces of the guiding bodies.

The present state of the science of hydro dynamics is unable to give rational expressions for these actions separately, we must be content with a summarisation.

For any given valve in properly constructed seat and box, we may in this way take the resistance as consisting of a part proportional to the velocity-head $\frac{u^2}{2g}$ and another part proportional to the velocity head $\frac{v^2}{2g}$ so that the total valve resistance will be measured by

$$\zeta \frac{u^2}{2g} = \zeta_1 \frac{u^2}{2g} + \zeta_2 \frac{v^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots (15)$$

where ζ is the co efficient of resistance of the valve, and ζ_1 and ζ_2 experimental co-efficients. Now

$$u a_1 = v\, c\, hq, \text{ or } v = u \frac{a_1}{c\, hq}$$

where c is the co efficient of contraction, hence (15) may be written :

$$\zeta \frac{u^2}{2g} = \frac{u^2}{2g} \left[\zeta_1 + \frac{\zeta_2}{c^2} \left(\frac{a_1}{hq} \right)^2 \right] \quad \ldots\ldots (16)$$

$$\text{or } \zeta = \zeta_1 + \frac{\zeta_2}{c^2} \left(\frac{a_1}{hq} \right)^2 \quad \ldots\ldots (16a)$$

For the valves here discussed $a_1 = \frac{\pi d^2}{4}$, and for valves guided on top $q = \pi d$. hence (16 *a*) becomes

$$\zeta = \zeta_1 + \frac{\zeta_2}{16 c^2} \left(\frac{d}{h} \right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Putting β for $\frac{\zeta_2}{16 c^2}$

$$\zeta = \zeta_1 + \beta \left(\frac{d}{h} \right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Section III.—Valve-Load or Valve Head when open.

Using the equation of continuity of flow we may similarly simplify equ. (12) for the valve head, which becomes :

$$P = G a_1 \frac{u^2}{2g} \left[\gamma + \left(\frac{d}{4 c h} \right)^2 \right], \quad \ldots\ldots\ldots\ldots (12a)$$

Equations (18) and (12 *a*) were first given by Bach in a treatise on Fire Engines in 1883 ; and they were experimentally tested as to their validity, and for the determination of their co efficients by him in 1884.

Part III.—Bach's Experiments.

In this section a succinct account of the results of Bach's experiments is given ; for the methods and apparatus used by him, see his treatise "Versuche über Ventilbeslatung u. Ventilwiderstand," Berlin, Julius Springer, 1884.

(*a*) Plate-valve, guided above, and plane under side (Fig. 11).

With this construction of valve were determined the interdependence of valve-load, valve resistance, and valve lift, and the relations between valve load and speed of water.

Another similar valve with larger seat area was used to determine the effect of this quantity.

The dimensions were $d = 50$ mm., $d_1 = 60$ mm., $d_2 = 90$ mm.

The variation of P with the lift h when the head under which water

flowed through was kept constant, for values of h between $\frac{d}{10}$ and $\frac{d}{4}$ may be expressed by

$$P = 1000\, a_1 \frac{u^2}{2g} \left[2.5 + \left(\frac{d}{4 \times 0.62h} \right)^2 \right] \quad \ldots(19)$$

f being $= 2.5$ and $c = 0.62$.

The variation of ζ with the lift h are expressed by equation (18); with $\zeta_1 = 0.55$ and $\beta = 0.15$; so that

$$\zeta = 0.55 + 0.15 \left(\frac{d}{h} \right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

for ordinary values of the lift between $\frac{d}{10}$ and $\frac{d}{4}$.

For a valve as in Fig. 11, but with the dimensions $d = 50$ **mm.** $d_1 = 74$ mm., $d_2 = 100$ mm. that is a broader seat-area:

$f = 5.15$ and $c = 0.605$.

so that (19) is

$$P = 1000\, a_1 \frac{u^2}{2g} \left[5.15 + \left(\frac{d}{4 \times 0.605\, h} \right)^2 \right] \quad \ldots\ldots(19a)$$

Also ζ_1 becomes 1.1 and β 0.155 so that (20) reads

$$\zeta = 1.1 + 0.155 \left(\frac{d}{h} \right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (20a)$$

(*b*) Valve concave downwards, guided above, as shown by Fig. 12, In this case in equation (12*a*) we get $f = 2.34$, and $c = 0.63$. For Fig. 11 we had

$f = 2.5$ and $c = 0.62$; so that we see that under similar conditions Fig. 11 requires a somewhat greater valve load than Fig. 12 contrary to what we should have expected.

With this valve ζ_1 becomes $= 0.65$ and $\beta = 0.132$, so that comparing with equation (20), it is seen that ζ_1 is greater and B less with a concave than with a flat valve. On the whole, the co-efficient of resistance ζ comes out smaller, for lifts from $\frac{d}{10}$ to $\frac{d}{4}$, for concave than for plane valves.

(*c*) Valve convex downwards; guided above.

Here the co-efficients in equation (12*a*) and (18) are almost identical, with those for a flat bottomed valve given in equations (19) and (20).

(*d*) Valves with guiding ribs below, as shown by Fig. 14.

Here the area through the valve seat is diminished 12.9 per cent. by the guide feathers.

If the number of ribs be i, then $p = \pi d - is$, so that equ. (12*a*) becomes:—

$$P = 1000\, a_1 \frac{u^2}{2g} \left[f + \left(\frac{a_1}{ch(\pi d - is)} \right)^2 \right]. \quad \ldots\ldots\ldots(21)$$

and the constants obtained from experiment were $f = 2.18 + c = 0.553$.

Also instead of equ. (18) we have here

$$\zeta = \zeta_1 + B \left(\frac{d^2}{(\pi d - is)h} \right)^2 \quad \ldots\ldots\ldots\ldots\ldots (22)$$

and for ordering lifts we may write $\zeta_1 = 1.35$ and $B = 1.7$.

SECTION 4.—PRACTICAL RULES.

Taking P as the valve load, or the force with which the opened valve must be loaded in order to maintain its equilibrium against the streaming fluid:—

d = dia of valve seat (v. Fig. 11)
$a = \pi d_2$ = area through do
h = lift of valve.
i = number of guide ribs, for clack valve
s = breadth of same (v. Fig. 4)
b = radial breadth of valve or seat facing $= \frac{1}{2}(d_2 - d)$ (Fig. 11)
u = speed of flow through a.
g = acceleration due to gravity $= 32.2$

ζ = co-effi. of resistance of the valve; so that head lost by resistance through the valve is $\zeta \frac{u^2}{2g}$, and ζ_1, β, γ, f, c, experimental co-efficients.

Then with the foot and the pound as units we have:

$$P = 62.4\, a \frac{u^2}{2g} \left[f + \left(\frac{d}{4\, ch} \right)^2 \right] \quad \text{.........} \quad (I)$$

$$P = 62.4 a \frac{u^2}{2g} \left[f + \left(\frac{a}{c(\pi d - is)h} \right)^2 \right] \text{.........} (II)$$

$$\zeta = \zeta_1 + \beta \left(\frac{d}{h} \right)^2 \quad \text{..........................} (III)$$

$$\zeta = \zeta_1 + \beta \left(\frac{d^2}{(\pi d - is)\, h} \right)^2 \quad \text{...... ....} (IV)$$

With the following values of constants:

1.—FOR PLATE VALVE AS IN FIG. 11.

In Equ. I. take $f = 2.5 + 19 \frac{b - 0.1d}{d}$ for breadths of b from $\frac{d}{10}$ to $\frac{d}{4}$; c = 0.60 to 0.63.

In Equ. III take $\zeta_1 = 0.55 + 4 \frac{b - 0.1d}{d}$ with b as above. β = 0.15 to 0.16.

Any deviation from a plane underside makes but little difference on the co-efficients; but it may be noted that ζ_1 is smaller for the valve in Fig. 12 and larger for Fig. 13 than for Fig. 11. The breadth of valve or seat face is much more influential than the form of the lower surface of the valve.

2.—FOR PLATE VALVES WITH GUIDE-RIBS, AS IN FIG. 14.

Use Equ. II with values of f and of c 10 p.c. less than for those for valves guided above, and Equ IV with values of ζ_1 from 0.8 to 1.6 greater than those in § 1 corresponding to a diminution of sectional area from 13 to 20 p. c.; and values of β from 1.7 to 1.75. The co-efficient of resistance ζ is of course very considerably greater with valves guided below.